NRES 746, Week 12, Lecture 1: Formula to Model (take-home)

Name: _____

Due Monday, Nov. 16. Work on paper; no code is needed. You may use the Week 12 web page and your notes.

Part 1: Counting parameters

You measure the height of sagebrush seedlings in quadrats. Quadrats are grouped into plots, and plots into sites; each quadrat has a shrub cover value (`cover`) and each site an elevation (`elev`). Different observers measured different quadrats.

1a. For each mixed model, count the fixed effects and the variance and correlation parameters (include the residual SD σ in that column), then the total.

Model formula	Fixed effects	Variance and correlation	Total
`height ~ cover + (1 | site)`			
`height ~ cover + (cover | site)`			
`height ~ cover + (cover || site)`			
`height ~ cover + (1 | site/plot)`			
`height ~ cover + elev + (cover + elev | site)`			
`height ~ cover + (1 | site) + (1 | observer)`			

1b. Which of the six models has nested random effects, and which has crossed random effects? What would make `site` and `observer` crossed rather than nested?

Part 2: From formula to equations

2a. Write the second model, `height ~ cover + (cover | site)`, as equations: the mean of an observation, the distribution of the residual, and the distribution of each site's random effects. Name every parameter you counted in 1a.

2b. For the nested model, `height ~ cover + (1 | site/plot)`, call the among-site SD τ_s, the among-plot (within-site) SD τ_p, and the residual SD σ. Fill in the covariance between the residual heights of two quadrats that are:

- in the same plot: _____
- in different plots at the same site: _____
- at different sites: _____

Part 3: Random slopes change the variance

Return to the drought-sensitivity model on the Week 12 web page:

$$\text{width} = (\mu_a + a_j) + (\mu_b + b_j)\,x + \varepsilon, \qquad (a_j, b_j) \sim \text{MVN}(0, G), \qquad \varepsilon \sim \text{Normal}(0, \sigma^2)$$

where x is precipitation in 10-cm units centered at 20 cm. Use $\tau_a = 0.15$, $\tau_b = 0.10$, $\rho = -0.6$, and $\sigma = 0.12$. Recall $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y) + 2\,\text{Cov}(X, Y)$ and $\text{Var}(cX) = c^2\,\text{Var}(X)$.

3a. Find the variance of one observation's width, for a stand you haven't sampled, as a function of x. Then evaluate it at $x = -1$ (a 10-cm winter) and $x = 1$ (a 30-cm winter).

3b. Ignore σ and look only at the spread among stands' expected widths, $\mathrm{Var}(a_j + b_j x)$. At what precipitation (in cm) is this spread smallest? What does that mean for the stands' precipitation response lines?

3c. In a sentence or two: what does $\rho = -0.6$ imply about which stands differ most in dry years?

Part 4: What the sampler sees

4a. The Stan program on the Week 12 web page fits the same model with $J = 12$ stands. Count the unknown quantities Stan samples (everything in the `parameters` block; for `L`, count only the values that are free to vary). How many does `lmer()` estimate for the same model?

4b. Why the difference? How would each count change with 50 stands?