NRES 746 – Week 2, Lecture 2: Limits and a Derivative

Name: _____

Instructions: Work alone for about 5 minutes. Then compare with a neighbor.

Part 1: What happens at the edges?

For each function below, use the "throw out the lower-order terms" trick to state what happens as $x \to 0$ and as $x \to \infty$. You don't need to show work, just the two limiting behaviors. We've already done the first two together in class!

1a. Michaelis-Menten: $f(x) = \dfrac{ax}{b + x}$

As $x \to 0$: _____ As $x \to \infty$: _____

1b. Ricker: $f(x) = ax\, e^{-bx}$

As $x \to 0$: _____ As $x \to \infty$: _____

1c. Logistic: $f(x) = \dfrac{1}{1 + e^{-bx}}$

As $x \to 0$: _____ As $x \to \infty$: _____

1d. Holling type III: $f(x) = \dfrac{ax^2}{b^2 + x^2}$

As $x \to 0$: _____ As $x \to \infty$: _____

Part 2: What kind of point is $x = 0$?

The logistic function, $f(x) = \dfrac{1}{1 + e^{-bx}}$, is the simplest sigmoid in the bestiary. It is symmetric around $x = 0$.

2a. Qualitatively, how would you characterize the derivative of the logistic function at x=0 relative to the derivative of this function at all other inputs (x)?

_____

_____

_____

2b. In Lab 1 you found that the Ricker function's derivative is zero at its peak. This makes it a 'critical point'. Is $f'(0)$ also zero at the logistic's midpoint (x=0)? If not, what kind of special point is $x = 0$ instead? (Hint: think about the curve's concavity just before and just after $x = 0$ – what would that imply about the *second* derivative at $x = 0$?)

_____

_____

_____

Optional, for fun! Rewrite the logistic function $f(x)$ as $(1 + e^{-bx})^{-1}$, then find $f'(x)$ using the chain rule (twice: once for the outer power, once for the inner exponential).

Reminders: $\dfrac{d}{dx}\,u^n = n\,u^{n-1}\,u'$ $\qquad$ $\dfrac{d}{dx}\,e^{g(x)} = g'(x)\,e^{g(x)}$

Show your work:

Now evaluate $f'(x)$ at $x = 0$.

$f'(0) =$ _____