

NRES 746, Week 8, Lecture 2: Curvature by Hand (optional take-home)

Name: _____

Optional practice; you don't need to turn it in. Try Parts 1 and 2 on paper before you open a laptop. Part 3 uses R.

Putting it in context

In Week 7 we saw that the curvature of the negative log-likelihood (NLL) at its minimum measures how much the data tell us about a parameter. For a parameter θ with MLE $\hat{\theta}$:

$$\text{SE}(\hat{\theta}) \approx \frac{1}{\sqrt{\text{NLL}''(\hat{\theta})}} \quad \text{approximate 95\% interval: } \hat{\theta} \pm 1.96 \text{ SE}$$

`optim(..., hessian = TRUE)` estimates $\text{NLL}''(\hat{\theta})$ numerically. Here you find it with calculus, for two models you already know. (Part 1 starts where the Newton's method take-home did, then goes on to the standard error.)

Part 1: Pika haypiles (Poisson)

The Week 7 pika data: active haypiles in six talus plots, 2, 5, 3, 0, 4, 4, modeled as Poisson with mean λ . Dropping terms that don't involve λ ,

$$\text{NLL}(\lambda) = n\lambda - \left(\sum_{i=1}^n y_i \right) \log \lambda$$

1a. Find $\text{NLL}'(\lambda)$ and $\text{NLL}''(\lambda)$.

1b. Evaluate $NLL''(\hat{\lambda})$ at $\hat{\lambda} = 3$ for the six plots, and again for 24 plots with the same mean (each count appearing four times). Give the SE for each.

6 plots: $NLL''(\hat{\lambda}) =$ _____, SE = _____ 24 plots: $NLL''(\hat{\lambda}) =$ _____, SE = _____

1c. Substitute $\hat{\lambda} = \bar{y} = \sum y_i/n$ into your NLL'' and simplify to get a general formula for $SE(\hat{\lambda})$ in terms of $\hat{\lambda}$ and n . What happens to the SE when n quadruples?

Part 2: Frog detection (binomial)

In Lab 5, Act 1, you pooled three wetlands and heard the frog on $K = 12$ of $N = 30$ visits, so $\hat{p} = 0.4$. Dropping the binomial coefficients,

$$\text{NLL}(p) = -K \log p - (N - K) \log(1 - p)$$

2a. Find $\text{NLL}''(p)$. (Watch the chain rule on $\log(1 - p)$.)

2b. Evaluate $\text{NLL}''(\hat{p})$ and the SE for your survey ($K = 12$, $N = 30$) and for your colleague's survey ($K = 120$, $N = 300$). Check: one of your curvatures should be 125.

3 wetlands: SE = _____ 30 wetlands: SE = _____

2c. Substitute $\hat{p} = K/N$ and simplify to get a general formula for $SE(\hat{p})$. Have you seen this formula before?

2d. The Week 7 Monday WOD (*Reading the Interval*) had 2 chytrid-positive frogs out of 20. Use your formula from 2c to build the approximate 95% interval, $\hat{p} \pm 1.96 SE$. What's wrong with it? Compare it with the range you read off the NLL curve in that WOD.

Interval: from _____ to _____

Part 3: Check it in R

Using AI: the coding question below is labeled with a recommendation. Writing this one yourself is good practice for the labs.

3a. (*Recommended: no AI.*) Write an NLL function for the frog data in 2b ($K = 12$, $N = 30$) using `dbinom()`, and minimize it with `optim(..., method = "Brent", lower = 0.001, upper = 0.999, hessian = TRUE)`. Does the Hessian match your answer to 2b?

`optim()` Hessian: _____

3b. Where did you use AI, if anywhere, and what did you have to check or fix? If you didn't use it, what was hardest?
